Classification of optimal binary subspace codes of length 8, constant dimension 4 and minimum distance 6

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The main result

Theorem ([HHK+], cf. [HK17])

$A_2(8,6;4) = 257$ *and all maximum codes are extended LMRD.*

That means that the maximum number of 4-dimensional spaces in $\mathbb{F}_2^8$ with pairwise intersection in at most a 1-dimensional space is 257,
or in terms of finte geometry, the maximum number of solids in $PG(7,2)$ mutually intersecting in at most a point is 257.

Any $(8,257,6;4)_2$ constant dimension code C is an extended LMRD, i.e.,
for a $(4 \times 4, 256, 3)_2$ maximum rank metric code M, the special solid $\bar{S} = \langle (0_{4\times4} \mid I_{4\times4}) \rangle$ and an arbitrary solid S intersecting $\bar{S}$ in a plane, the two possibilities for C (up to symmetry) are

1. $\{\langle (I_{4\times4} \mid B) \rangle \mid B \in M\} \cup \{\bar{S}\}$ and
2. $\{\langle (I_{4\times4} \mid B) \rangle \mid B \in M\} \cup \{S\}$.

Other optimal cases

Theorem ([HHK$^+$], cf. [HK17])
$A_2(8,6;4) = 257$ *and all maximum codes are extended LMRD.*

The other optimal cases (nontrivial and no partial spread) are:

1. $A_2(6,4;3) = 77$ with 5 maximum codes [HKK15] and
2. $A_2(13,4;3) = 1597245$ (no classification known) [BEÖ$^+$16]

Outline of the proof I

Denote $\bar{P}$ and $\bar{H}$ a nonincident point hyperplane pair.

By counting and symmetry:

1. Each point and hyperplane is incident to ≤ 17 codewords.
2. If $\#C \geq 256$ then $\bar{H}$ and $\bar{P}$ are together incident to ≥ 31 codewords.
3. Then $\bar{H}$ contains ≥ 16 codewords.
4. Then the codewords in $\bar{H}$ are orthogonal $(7, N, 6; 3)_2$ cdcs with $\#N \geq 16 \ldots$
5. ... which are classified: $(7, 17, 6; 3)_2$ (715), $(7, 16, 6; 3)_2$ (14445) [HKK16]

Outline of the proof II

Phase 1:
Exclude $(7, N, 6; 3)_2$ cdcs with $\#N \geq 16$ embedded in $\mathbb{F}_2^8$ or directly in $\mathbb{F}_2^7$ via (integer) linear programming

Phase 2:
Extend reminaing N-configurations to 31-configurations via an all-clique problem and symmetry

Phase 3:
Exclude 31-configurations via (integer) linear programming in $\mathbb{F}_2^8$
$\Rightarrow A_2(8, 6; 4) = 257$

Phase 4:
Gain structural insight via integer linear programming of 31-configurations that are subset of maximum code
$\Rightarrow$ all maximum codes are extended LMRD

Thank you for your attention I

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  Classification of optimal binary subspace codes of length 8, constant dimension 4 and minimum distance 6.
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  A new upper bound for subspace codes.
  arXiv preprint 1703.08712, 2017.

Thank you for your attention II

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Classification of large partial plane spreads in PG(6, 2) and related combinatorial objects.
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