Classification of optimal binary subspace codes of length 8, constant dimension 4 and minimum distance 6

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The main result

Theorem ([HHK$^+$], cf. [HK17])

$A_2(8, 6; 4) = 257$ and all maximum codes are extended LMRD.

That means that the maximum number of 4-dimensional spaces in $\mathbb{F}_2^8$ with pairwise intersection in at most a 1-dimensional space is 257,

or in terms of finite geometry, the maximum number of solids in $\text{PG}(7, 2)$ mutually intersecting in at most a point is 257.

Any $(8, 257, 6; 4)_2$ constant dimension code $C$ is an extended LMRD, i.e.,

for a $(4 \times 4, 256, 3)_2$ maximum rank metric code $M$, the special solid $\bar{S} = \langle(0_{4\times4} | I_{4\times4})\rangle$ and an arbitrary solid $S$ intersecting $\bar{S}$ in a plane, the two possibilities for $C$ (up to symmetry) are

1. $\{\langle(I_{4\times4} | B)\rangle | B \in M\} \cup \{\bar{S}\}$ and

2. $\{\langle(I_{4\times4} | B)\rangle | B \in M\} \cup \{S\}$. 
Other optimal cases

Theorem ([HHK$^+$], cf. [HK17])

\[ A_2(8, 6; 4) = 257 \text{ and all maximum codes are extended LMRD.} \]

The other optimal cases (nontrivial and no partial spread) are:

1. \[ A_2(6, 4; 3) = 77 \text{ with 5 maximum codes [HKK15] and} \]
2. \[ A_2(13, 4; 3) = 1597245 \text{ (no classification known) [BEÖ}^+16] \]
Outline of the proof I

Denote $\bar{P}$ and $\bar{H}$ a nonincident point hyperplane pair.

By counting and symmetry:

1. Each point and hyperplane is incident to $\leq 17$ codewords.
2. If $\#C \geq 256$ then $\bar{H}$ and $\bar{P}$ are together incident to $\geq 31$ codewords.
3. Then $\bar{H}$ contains $\geq 16$ codewords.
4. Then the codewords in $\bar{H}$ are orthogonal $(7, N, 6; 3)_2$ cdcs with $\#N \geq 16$ . . .
5. . . . which are classified: $(7, 17, 6; 3)_2 (715)$, $(7, 16, 6; 3)_2 (14445)$ [HKK16]
Outline of the proof II

Phase 1:
Exclude \((7, N, 6; 3)_2\) cdcs with \(\#N \geq 16\) embedded in \(\mathbb{F}_2^8\) or directly in \(\mathbb{F}_2^7\) via (integer) linear programming

Phase 2:
Extend remaining \(N\)-configurations to 31-configurations via an all-clique problem and symmetry

Phase 3:
Exclude 31-configurations via (integer) linear programming in \(\mathbb{F}_2^8\)
\[\Rightarrow A_2(8, 6; 4) = 257\]

Phase 4:
Gain structural insight via integer linear programming of 31-configurations that are subset of maximum code
\[\Rightarrow \text{all maximum codes are extended LMRD}\]
Thank you for your attention


Thomas Honold, Michael Kiermaier, and Sascha Kurz. 
Optimal binary subspace codes of length 6, constant dimension 3 and minimum subspace distance 4. 

Thomas Honold, Michael Kiermaier, and Sascha Kurz. 
Classification of large partial plane spreads in PG(6, 2) and related combinatorial objects. 